Ques Transform the axes inclined at 30° to the orignal axes for the equation.
$$x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2.$$

Soln. $x = x_1 \cos 30^\circ - y_1 \sin 30^\circ$
$$= \frac{x_1\sqrt{3}}{2} - \frac{y_1}{2} = \frac{x_1\sqrt{3} - y_1}{2} \quad \text{--- (1)}$$

$y = x_1 \sin 30^\circ + y_1 \cos 30^\circ$
$$= \frac{x_1}{2} + \frac{y_1\sqrt{3}}{2} = \frac{x_1 + y_1\sqrt{3}}{2} \quad \text{--- (2)}$$

Substituting eqn (1) & (2) in given eqn.
we get
$$\left(\frac{x_1\sqrt{3} - y_1}{2}\right)^2 + 2\sqrt{3}\left(\frac{x_1\sqrt{3} - y_1}{2}\right)\left(\frac{x_1 + y_1\sqrt{3}}{2}\right) - \left(\frac{x_1 + y_1\sqrt{3}}{2}\right)^2 = 2a^2$$

$$\Rightarrow \quad x_1^2 - y_1^2 = a^2$$
which is the required eqn referred to new axes.

Ques If (x, y) and (x_1, y_1) be the co-ordinates of the same point reffered to two sets of rectangular axes with the same origin & if $ux + vy$ where 'u' & 'v' are independent of x & y. becomes $u_1x_1 + v_1y_1$, show that $u^2 + v^2 = u_1^2 + v_1^2$

Soln Let the angle between new axes and orignal axes be θ

$\Rightarrow x = x_1\cos\theta - y_1\sin\theta$, $y = x_1\sin\theta + y_1\cos\theta$

$\Rightarrow ux + vy = u(x_1\cos\theta - y_1\sin\theta) + v(x_1\sin\theta + y_1\cos\theta)$
$= (u\cos\theta + v\sin\theta)x_1 + (v\cos\theta - u\sin\theta)y_1$

$\Rightarrow u_1 = (u\cos\theta + v\sin\theta)$ & $v_1 = (v\cos\theta - u\sin\theta)$

$ux + vy = u_1x_1 + v_1y_1$

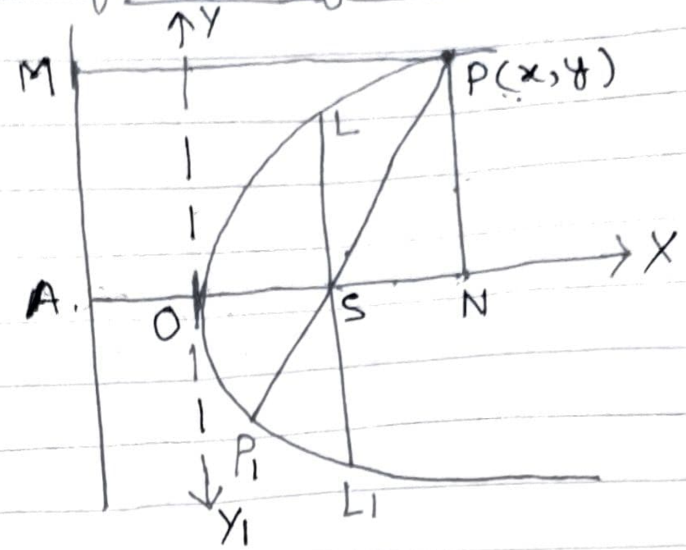
Proved

Parabola

Conic Section – The locus of a point which moves such that its distance from a fixed point bears a constant ratio to its distance from a fixed straight line is called a conic section.
The fixed point is called focus
The straight line is called directrix
The constant ratio is called eccentricity denoted by 'e'.

If $e<1$ then the conic section is ellipse.
If $e=1$ then the conic section is parabola
If $e>1$ then the conic section is hyperbola

General equation of Parabola



Let $P(x,y)$ be any point on the curve & S be the focus (α, β) and PM be the $\perp$ on directrix. Then by definition.

Let the eqn of directrix be $ax+by+c=0$

$$\frac{SP}{PM} = e = 1 \Rightarrow SP^2 = PM^2$$

$$(x-\alpha)^2 + (y-\beta)^2 = \frac{(ax+by+c)^2}{a^2+b^2}$$

On simplification we get

$$(bx-ay)^2 = 2x\left[ac + \alpha(a^2+b^2)\right] + 2y\left(bc + \beta(a^2+b^2)\right) + \left[c^2 - (a^2+b^2)(\alpha^2+\beta^2)\right]$$

which is the general eqn of parabola in second degree.

Standard Eqn of Parabola

Let 'S' be the focus and MA be the directrix. From S draw $SA \perp MA$ and let 'O' be the middle point of AS.

$\frac{OS}{OA} = 1$, ∴ 'O' is also vertex of parabola

Let $OS = a$. Take 'O' as origin and OSX as x-axis and $OY \perp OSX$ as y-axis

Let P be any point on the curve with co-ordinates (x, y). Draw $PN \perp OX$ & $PM \perp AM$.

∴ $ON = x$, $PN = y$

Also $SN = ON - OS$

$\Rightarrow SN = (x - a)$

$AN = ON + OA = ON + OS$

$\Rightarrow AN = (x + a)$

Now $SP = PM \Rightarrow SP^2 = PM^2$

$\Rightarrow SN^2 + NP^2 = AN^2$

$\Rightarrow (x - a)^2 + y^2 = (x + a)^2$

$\Rightarrow y^2 = (x + a)^2 - (x - a)^2$

$= 4ax$

$\Rightarrow y^2 = 4ax$ is the required standard eqn of the parabola.